

MIT-CTP-2923  
hep-ph/9911329

## THE $\mu$ -PARAMETER OF SUPERSYMMETRY

NIR POLONSKY

*Center for Theoretical Physics, Massachusetts Institute of Technology,  
77 Massachusetts Avenue, Cambridge, MA 02139-4307 USA  
E-mail: nirp@ctpgreen.mit.edu*

### ABSTRACT

The Higgsino mass, or equivalently the  $\mu$ -parameter, plays an essential role in determining the phenomenology of any supersymmetric model. Particularly, the size of the supersymmetry conserving  $\mu$ -parameter must be correlated with the size of the soft supersymmetry breaking parameters. The source of this correlation in the underlying ultra-violet theory is one of the mysteries of supersymmetry model building. The puzzle and the various possibilities for its resolution are reviewed, stressing both phenomenological and theoretical aspects. New proposals in the context of supergravity and gauge-mediation frameworks for the soft supersymmetry breaking parameters are examined in some detail.

Talk presented at SSS-99  
Supersymmetry, Supergravity, and Superstring  
Seoul National University, Korea, June 23 – 27, 1999

## 1 Introduction

The minimal supersymmetric extension (MSSM) of the Standard Model (SM) of electroweak and strong interactions, defined here by its minimal matter content, is described by the superpotential

$$W = -\mu H_U H_D - h_U H_U Q U + h_D H_D Q D + h_E H_D L E, \quad (1)$$

where  $H_U$  (with hypercharge  $Y = +1$ ) and  $H_D$  ( $Y = -1$ ) are the two Higgs doublets required by the holomorphicity of  $W$  and by consistency. Also, we employ the usual notation for the quark doublet ( $Q$ ) and singlets ( $U$  and  $D$ ) and for the lepton doublet ( $L$ ) and singlet ( $E$ ). (Color, isospin and generation indices are suppressed. For a tabulation of the different sign conventions and for a general review, see Ref. 1.) The field theory Lagrangian is given by the superspace integration  $\mathcal{L} \sim \int d^2\theta W$ . Though the superpotential (1) is that of the MSSM, it provides the core superpotential terms of any extension of the MSSM. (For a caveat, see Ref. 2.) In particular, the MSSM, or any of its extensions, contains a (supersymmetric) Higgs mixing term  $\mu H_U H_D$ , or more generally, a Yukawa “mass” term involving a singlet or background superfield  $X$ ,  $W \sim \mu H_U H_D + \dots \rightarrow W \sim \lambda X^n H_U H_D + \dots$  and  $\mu \rightarrow \lambda \langle X \rangle^n$  ( $n$  is determined by the dimensionality of the coupling  $\lambda$ ,  $n = 0$  for  $\lambda = \mu$ ;  $n = 1$  if  $\lambda$  is a Yukawa term; and  $n > 1$  in the case of non-renormalizable couplings.).

While the singlet interpretation will prove to be a convenient tool in the discussion below, let us first assume a dimensionful parameter  $\mu$  ( $n = 0$ ) in the effective theory which describes the regime between the weak scale  $\Lambda_W$  and a few hundred GeV. This assumption, which we are about to justify, already hints at the puzzle we are about to address: The natural choice of a scale for a dimensionful superpotential parameter is the scale of the ultra-violet theory, let it be the (reduced) Planck mass  $\Lambda_P \sim 2 \times 10^{18}$  GeV, the unification scale  $\Lambda_U \sim 10^{16}$  GeV, or a messenger scale in a gauge-mediation framework  $\Lambda_M \gtrsim 10^5$  GeV. Therefore, unless  $\mu = 0$  (we return to and exclude this possibility below) one expects  $|\mu| \gg \Lambda_W$ . Nevertheless, a viable phenomenology requires  $|\mu| \simeq \Lambda_W$ , or more correctly,  $|\mu| \simeq \Lambda_{\text{SSB}}$ , where  $\Lambda_{\text{SSB}} \simeq (4\pi/h_t)\Lambda_W$  is the scale of the soft supersymmetry breaking (SSB) parameters which is constrained from above by the stability of the weak scale (*i.e.*, the hierarchy problem), and  $h_t \equiv h_{U_3}$  is the  $t$ -quark Yukawa coupling. This implies that all of the dimensionful parameters of the supersymmetric extension have a similar origin, even though from the low-energy point of view they are of a very different nature. This puzzle was first formulated in Ref. 3, and it suggests that the Higgs fields may be distinguished from all other matter in the way in which they communicate with heavy and supersymmetry breaking fields.

Supersymmetry provides technically natural solutions to the hierarchy problem, *i.e.*, the smallness of the weak scale  $\Lambda_W \simeq 0$  (in Planckian units). One can further understand, using field-theory tools, spontaneous supersymmetry breaking in a hidden sector at a given scale  $\Lambda_{\text{SUSY}}$  as well as the propagation of the information to the observable sector and the generation of the SSB parameter scale  $\Lambda_{\text{SSB}}$ , which in turn sets  $\Lambda_W$ . However, for the relation  $\Lambda_{\text{SSB}} \simeq \Lambda_W$  to hold, we must also understand the correlation  $\mu \simeq \Lambda_{\text{SSB}}$ , as we will attempt to do below. Furthermore, this tension implies that  $\mu$  cannot be given by the ultra-violet cut-off scale but instead, it parameterizes physics at that scale which is responsible for its smallness and its association with the SSB scale. This will lead us to treat  $\mu$  as a spurion degree of freedom below, and adopt and generalize the “singlet presentation” given above, but more importantly, it offers a potential benefit – another door to the ultra-violet regime.

We must first argue, however, for the above conclusions in more detail. This is done in Sec. 2. In Sec. 3 we attempt to understand the origin of the correlation between  $\mu$  and  $\Lambda_{\text{SSB}}$ . We describe the various possibilities in an effective operator language and comment on their viability in the different frameworks. The various manifestations of the  $\mu$ -problem are also discussed and compared. We then turn in Sec. 4 to describe two specific realizations that were proposed recently in the context of supergravity and gauge-mediation frameworks for the SSB parameters. In Sec. 5 we comment on a generalization of the  $\mu$ -parameter to lepton number violating theories. We conclude in Sec. 6.

## 2 Some Phenomenology

The (phenomenological) scale correlation between  $\mu$  and the SSB parameters can be derived from various considerations. For example, consider the scalar potential given by

$$V = \sum \left| \frac{\partial W}{\partial \Phi} \right|_{\phi}^2 + \text{gauge } D\text{-terms} + V_{\text{SSB}}, \quad (2)$$

where the summation is over all chiral superfields  $\Phi = H_U, H_D, Q, \dots$ ,  $\Phi = \phi + \theta\psi + \theta^2 F$  ( $\theta$  is the superspace coordinate), and the gauge  $D$ -terms  $\sim (g^2/2) \sum_{a,i} |\phi_i T^a \phi_i^\dagger|^2$ , as usual. The SSB terms contained in  $V_{\text{SSB}}$  are those terms that are mediated in the SM visible sector by some messenger interactions which communicate between the visible and some other hidden sector in which supersymmetry is broken spontaneously. (It is said to be hidden because by construction its interactions with the visible sector are

strongly restricted, and in particular, could not be tree-level renormalizable interactions.)

The scalar potential (2) includes the (tree-level improved) Higgs potential

$$V(H) = (m_{H_U}^2 + \mu^2)|H_U|^2 + (m_{H_D}^2 + \mu^2)|H_D|^2 - m_{H_{UD}}^2(H_U H_D + \text{H.c.}) + \frac{g_Y^2 + g^2}{8} (|H_U|^2 - |H_D|^2)^2 \quad (3)$$

where  $g_Y$  ( $g$ ) is the SM hypercharge ( $SU(2)$ ) coupling. Our notation does not distinguish a chiral superfield  $\Phi$  and its scalar field component  $\phi$ , we assume real parameters, and  $m_{H_U}^2$ ,  $m_{H_D}^2$ , and  $m_{H_{UD}}^2 \equiv B\mu$  are SSB parameters. Clearly,  $\text{Max}[\mu^2, m_{H_U}^2, m_{H_D}^2, m_{H_{UD}}^2]$  controls the scale of the potential and hence the realization and scale of electroweak symmetry breaking (EWSB). The puzzle discussed above can now be rephrased: Why is the supersymmetry conserving Higgs mixing parameter in the superpotential,  $\mu$ , of the same order of magnitude as the supersymmetry breaking mixing parameter in the scalar potential,  $m_{H_{UD}}$ ?

Returning to the Higgs potential, it contains a flat direction  $m_{H_U}^2 + m_{H_D}^2 + 2\mu^2 - 2m_{H_{UD}}^2 = 0$  (and hence, a light Higgs boson), and is consistent with electroweak symmetry breaking iff  $(m_{H_U}^2 + \mu^2)(m_{H_D}^2 + \mu^2) < |m_{H_{UD}}^2|^2$ . The latter condition is achieved in typical models radiatively as a result of large Yukawa quantum corrections (and hence, radiative symmetry breaking (RSB)). Once the symmetry is broken then the weak scale, given by the  $Z$  mass, can be written in terms of  $\mu$  and the SSB parameters, or equivalently

$$\mu^2 = \frac{m_{H_D}^2 - m_{H_U}^2 \tan^2 \beta}{\tan^2 \beta - 1} - \frac{1}{2}m_Z^2, \quad (4)$$

where  $\tan \beta = \langle H_U^0 \rangle / \langle H_D^0 \rangle$  is the usual ratio of Higgs vacuum expectation values (VEVs). By observation,  $|\mu|$  is given by a cancellation between the SSB parameters and the experimentally determined weak scale, and hence, in order to avoid fine tuning it must fall within this range. (The weak and SSB scales themselves are correlated by the cancellation of quadratic divergences and stability conditions.) Diagonalization of the Higgs boson mass matrices, including EWSB effects, in the  $\mu \gg \Lambda_W$  limit, exhibits a decoupling of one Higgs (boson) doublet, which effectively does not participate in EWSB, and of the Higgsinos, all with mass  $\sim \mu$ . The remaining light doublet boson which is responsible for EWSB (and which contains the flat direction, and hence a light Higgs boson) has no fermion superpartner. The resulting quadratically divergent quantum correction again put an upper bound on the size of  $\mu$ . (In fact, one can use this observation in order to calculate (leading) quantum correction to the light-Higgs mass.<sup>4</sup>)

It is worth commenting on the issue of fine tuning. No objective definition for fine tuning exists but clearly any sensible theory must avoid large cancellations – unless they are a remnant of correlations or symmetries in the ultra-violet theory (which may be mistaken by an infra-red observer for fine tuning). Applying this mild criterion to electroweak symmetry breaking Eq. (4) and recalling that the symmetry breaking is encoded in the (one-loop renormalized) SSB parameters, imply that the ultra-violet theory must correlate the SSB parameters and the effective low-energy  $\mu$ . When solving Eq. (4) in a given model, one often finds a *de facto* correlation between  $\mu$  and the gluino mass  $M_{\tilde{g}}$  which indirectly controls the renormalization of the SSB parameter  $m_{H_U}^2$ . Hence, in many cases correct phrasing of the fine-tuning issues need to be in terms of  $|\mu/M_{\tilde{g}}|$ , or more generally  $|\mu/\Lambda_{\text{SSB}}|$ , rather than in terms of  $|\mu/m_Z|$  (as is often done). To reiterate, the issue is not the value of a precisely measured infra-red parameter  $m_Z$ , but the understanding of the correlations among the ultra-violet parameters which produce this value. These correlations, of course, become more numerically constraining and, therefore, more difficult to envision or formulate as  $\mu \sim \Lambda_{\text{SSB}} \rightarrow \infty$  decouple. (This limit corresponds to the restoration of the original hierarchy problem.)

Returning to the scalar potential (2), it contains other terms involving  $\mu$  which arise from the cross terms in  $|\partial W/\partial \Phi|^2$ . These terms constitute (non-holomorphic) tri-linear Higgs-left-right (LR) couplings (where  $\Phi_L = Q$ ,  $L$  are the  $SU(2)$  doublets and  $\Phi_R = U, D, E$  are the  $SU(2)$  singlets). After EWSB they provide chirality violating off-diagonal LR entries in the sfermion  $\tilde{f}$  mass-squared matrices,

$$\overline{m}_f^2 = \begin{pmatrix} m_{LL}^2 & m_{LR}^2 \\ m_{LR}^2 & m_{RR}^2 \end{pmatrix}, \quad (5)$$

where the LR mixing mass,

$$m_{LR}^2 = m_f (A_f - \mu \tan \beta) \quad [\text{or } m_f (A_f - \mu/\tan \beta)], \quad (6)$$

includes also SSB tri-linear  $A$ -parameters, which are implicitly assumed to be proportional to the Yukawa coupling  $\hat{A}_f = y_f A_f$ , which, in turn, is factored out, and  $\mu \tan \beta$  ( $\mu/\tan \beta$ ) terms appear in the down-squark and slepton mass matrices (up-squark mass matrix). The requirement of a stable minimum and the positivity of the determinant constrain  $|\mu|$  from above as well. In addition, even if the EWSB minimum is stable, it may be only a local minimum of the whole scalar potential (2) while a charged and/or colored field acquires a non-vanishing VEV along a direction in field space that corresponds to a deeper (global) minimum, or along a flat direction. These considerations constrain

the possible relations between the different parameters. For example, for  $h_t \sim 1$  the constraint

$$(A_t \pm \mu)^2 \leq 2(m_{Q_3}^2 + m_{U_3}^2) \quad (7)$$

is found.<sup>5</sup> (The undetermined sign on the left-hand side is given by  $\text{sign}(A/\mu)$ .)

All of the above establishes our previous assertion that a viable phenomenology requires  $\mu \sim \Lambda_{\text{SSB}} \sim (1 - 10) \times \Lambda_W$ . The crucial role of  $\mu$  in determining the phenomenology of the models, however, is apparent in many other cases. In particular, the  $\mu$  parameter also dominates (or contributes significantly to) many “supersymmetric quantum corrections” either via the chirality flipping LR sfermion mass squared or the Higgsino mass term. These include all the chirality violating magnetic moment operators ( $b \rightarrow s\gamma$  amplitude, the anomalous muon magnetic moment  $a_\mu$ , and finite corrections to fermion masses) as well as radiative corrections to the light Higgs boson mass (which are enhanced in the case of large LR stop mixing). Most surprisingly, it also controls the one-loop threshold corrections to gauge coupling unification predictions.<sup>6</sup> These, together with its role in determining the neutralino, chargino, and Higgs spectrum and couplings, imply that  $\mu$  will be known if supersymmetry is discovered and established experimentally.

This leads to another question raised above: Is there sufficient evidence (when assuming supersymmetry) that  $\mu \neq 0$ ? If it were zero it would resolve the most difficult part of the puzzle, *i.e.*, why  $\mu \simeq \Lambda_{\text{SSB}}$ , while  $\mu = 0$  can be understood in terms of an enhanced (unbroken) symmetry (e.g. an  $U(1)_R$  symmetry under which the superpotential has non-trivial charge<sup>7</sup>). Setting a lower bound on a parameter rather than a mass of a physical state is not trivial. However, it can be done in this case by consideration of chargino pair production. By observation, the (tree-level) chargino mass matrix,

$$m_{\tilde{\chi}^\pm} = \begin{pmatrix} M_2 & \sqrt{2}m_W \sin \beta \\ \sqrt{2}m_W \cos \beta & -\mu \end{pmatrix}, \quad (8)$$

contains charginos degenerate in mass with the  $W$  bosons for  $\mu = M_2 = 0$  and  $\tan \beta = 1$ , where  $M_2$  is a SSB Wino mass parameter. Such charginos would have escaped detection at the  $Z$ -pole (LEPI), but would be pair-produced in abundance at the  $WW$  threshold (LEPII). They would then decay to a massless neutralino and a  $W$  boson, hence, imitating  $WW$  production. However, no such deviations from SM predictions were observed, excluding this possibility. In practice, one needs to consider neutralino production as well. Surprisingly, a small region of parameter space with  $\mu \simeq M_2 \simeq \mathcal{O}(\text{GeV})$  survived a careful analysis of all  $Z$ -pole data<sup>8</sup> even when including radiative corrections and off-peak data. While neutralino production above the  $Z$ -peak is

complicated by  $t$ -channel sneutrino exchange,<sup>9</sup> the  $WW$ -threshold constraints (even when applied to one on-shell and one slightly off-shell chargino) provide relatively a model-independent constraint  $|\mu| \gtrsim \Lambda_W$ , as suggested in Ref. 8.

Throughout the discussion it is assumed for simplicity that  $\mu$  is a real parameter and we conveniently identify its ultra-violet and infra-red values. In general  $\mu$  is a complex parameter that carries a physical phase. While it has been argued recently<sup>10</sup> that such a phase could be substantial due to accidental cancellations among various contributions to, e.g., the neutron dipole moment, we will not entertain this possibility here. Phases are also less constrained in the  $U(1)_R$  limit<sup>11</sup>  $|\mu| \ll m_{\tilde{f}}$ , but again we do not consider this option here. In practice, our discussion is mostly independent of any assumption about the phases. The sign of the real  $\mu$  is still physical as it affects the various quantum corrections and phenomena discussed above, but again is of no concern in the discussion below.  $\text{Sign}(|\mu|)$  is a renormalization group invariant. The magnitude  $|\mu|$ , on the other hand, is subject to wave function renormalization so that the superpotential parameter  $\mu$  is renormalized only in proportion to itself, and only by electroweak and Yukawa couplings. In particular, it cannot mix via renormalization with the SSB parameters. Since our discussion here is mostly qualitative we can safely ignore these effects which are typically small.

Equipped with (rough) upper and lower bounds on the absolute value of the  $\mu$ -parameter, which suggest that it is correlated with the SSB parameters, we turn to a discussion of the possible origin of this correlation.

### 3 Operator Analysis

Hereafter we adopt the point of view that in the ultra-violet (UV) theory, given its symmetries,  $\mu_{UV} = 0$ , while  $\mu_{IR} \simeq \Lambda_{SSB}$  is generated by terms which appear once supersymmetry is spontaneously broken in some hidden sector of the theory. Such terms break the relevant symmetries either explicitly or spontaneously in the effective infra-red (IR) theory. (As discussed above,  $\mu_{IR}$  renormalization cannot change its order of magnitude and is ignored.) The parameter  $\mu_{IR}$  then implicitly carries information on the symmetries of the UV theory and on their violation by supersymmetry breaking. It must also contain some information on the supersymmetry breaking fields. This is seen most clearly once we return to viewing  $\mu$  as a VEV of (a physical or an auxiliary component of) a (background) singlet superfield  $X$  which may or may not participate directly in supersymmetry breaking. More generally, let us denote by  $X$  a singlet object that can couple to the holomorphic bilinear  $H_1 H_2$ , leaving aside for the moment its identity or our definition of a singlet

object. One can describe the superpotential and SSB Higgs mixing parameters in terms of effective  $F$ -term ( $\int d^2\theta$ ) and  $D$ -term ( $\int d^2\theta d^2\bar{\theta}$ ) operators

$$\begin{aligned} \text{SUSY Higgs mixing } (\mu) : & \quad a \int d^2\theta H_1 H_2 \left[ \frac{X^n}{M^{n-1}} \right] \\ & \quad \text{and } a' \int d^2\theta d^2\bar{\theta} H_1 H_2 \left[ 1 + \frac{X^\dagger}{M} + \dots \right] + \text{H.c.}, \quad (9) \\ \text{SSB Higgs mixing } (m_{H_{UD}}^2) : & \quad b \int d^2\theta H_1 H_2 \left[ X + \frac{X^2}{M} + \dots \right] \\ & \quad \text{and } b' \int d^2\theta d^2\bar{\theta} H_1 H_2 \left[ \frac{X^\dagger X}{M^2} + \dots \right] + \text{H.c.}, \quad (10) \end{aligned}$$

with the usual conventions  $\int d^2\theta X = F_X$  in Eq. (10) and  $\int d^2\bar{\theta} X^\dagger = F_X^\dagger$  in Eqs. (9) and (10). The scale  $M$  parameterizes the mediating interactions and is of the same order of magnitude as the scale of the mediation of supersymmetry breaking from the hidden to the visible sector  $\Lambda_{\text{SUSY}}$ . It may be a true parameter or a background field itself  $M = \langle Y \rangle$ . We wrote explicitly all the higher order operators in the  $F$ -term integral in Eq. (9) since only the scalar component of  $X$  contributes in this case, and for sufficiently large  $X$  higher order operators could be considered. Similarly, the leading  $D$ -term operator in (9) can also be identified with  $\langle X \rangle / M \sim 1$ . Otherwise, higher order operators are denoted by  $\dots$ .

To reach a resolution of the puzzle one needs to obtain the same order of magnitude in both operators (9) and (10). (Note that in the latter case there could be contributions from non-singlet fields  $ZZ^\dagger$ .) We now need to understand the nature of  $X$ , its value  $\langle X \rangle$ , and its relation to supersymmetry breaking, *i.e.*,  $\langle F_X \rangle$ . We also need to understand the expectation for the coefficients, which is crucial in distinguishing realizations in the different frameworks. Finally, one may want to understand the symmetries which forbid/allow the various operators. Note that the  $\mu$  term breaks, in general, both  $U(1)$  Peccei-Quinn and  $R$  symmetries while the SSB mixing term breaks, in general, only the Peccei-Quinn symmetry. Both symmetries could play a role in providing the appropriate selection rules for the operators. In particular,  $R$  symmetry is a powerful tool as it does not commute with supergravity ( $R(\theta) = -1$ ) and all hidden and visible sectors are charged under it and thus the resulting selection rules can be applied to also mixed hidden-visible operators.

Let us begin and examine the different operators. Assume that the operators are tree-level operators and that the dimensionless coefficients are generic  $\mathcal{O}(1)$  coefficients. This corresponds to the “classical” supergravity



framework (*i.e.*, tree-level gravity mediation of the SSB parameters in the visible sector via Planck suppressed operators), in which case  $M \sim \Lambda_P$  and  $\Lambda_{SSB}$  is simply given by the gravitino mass  $m_{3/2} = \Lambda_{SUSY}^2 / \sqrt{3} \Lambda_P$  (assuming cancellation of the cosmological constant), which, in turn, has to be fixed. A primordial  $\mu$  term ( $n = 0$ ) could be forbidden by  $R$  symmetry<sup>12,13</sup> if  $R(H_U H_D) \neq R(W) = 2$ , for example. It could also be forbidden by a variety of other symmetries such as a Peccei-Quinn ( $PQ$ ) symmetry<sup>3,14</sup> under which  $PQ(H_D) + PQ(QD) = 0$  and  $PQ(H_U) + PQ(QU) = 0$ , a discrete  $Z_3$  (or higher) symmetry<sup>15</sup>, a flavor<sup>16</sup>  $U(1)$ , an extended gauge structure,<sup>17</sup> or a grand-unified (GUT) symmetry.<sup>18,19</sup> (It could also be forbidden by modular invariance in string theory, in which case the effective  $\mu$  is a function of the moduli fields.<sup>20</sup>)

The next class of operators is the  $n = 1$  Yukawa operators in the superpotential which is relevant for our purposes iff  $\langle X \rangle \simeq m_{3/2}$ . The most advertised realization of this is the next to minimal MSSM (NMSSM) in which the spectrum is extended by a gauge singlet and a discrete  $Z_3$  symmetry is imposed.<sup>15</sup> The singlet survives to low energies and a mechanism similar to the one leading radiatively to EWSB with  $\langle H_{U,D} \rangle \sim \Lambda_{SSB}$  now generates  $\langle X \rangle \sim \Lambda_{SSB}$  (where the potential is stabilized by a  $X^3$  term in the NMSSM superpotential). While the supersymmetry conserving VEV of  $X$  induces  $\mu$ , the supersymmetry breaking VEV of the auxiliary component  $F_X \sim X^2$  provides the SSB mixing term (the first  $F$ -term operator in Eq. 10). It could also arise from higher order terms in the first ( $F$ -term) integral in Eq. (10), which effectively correspond to a SSB term  $\sim AX H_U H_D$  in  $V_{SSB}$  if  $X^2 \rightarrow X_0 X$  and  $X_0$  is a supersymmetry breaking (hidden-sector) field with  $F_{X_0} \sim \Lambda_{SUSY}^2 \sim m_{3/2} \Lambda_P$ . While the extended spectrum is consistent with all phenomenological constraints, the spontaneously broken  $Z_3$  symmetry leads to post-inflationary domain-wall problem which disfavors this construction.<sup>15</sup> A distinctive alternative is to gauge the extra  $U(1)$  which appears<sup>17</sup> (with  $Q(X) = -Q(H_U H_D)$ ), in which case the  $Z_3$  is a harmless discrete subgroup.  $X$  in this case is only a SM singlet and not a gauge singlet. Aside from the presence of an additional neutral massive gauge boson, it is distinguished from the usual NMSSM by the form of the quartic potential of the singlet that now arises from the (extended)  $D$ -terms in (2). We will explore this option below in some detail, but in a different context.

A different approach is to consider a background field  $X$  that is decoupled from the sub-TeV theory but its VEV is sufficiently small so that terms in the low-energy theory could be proportional to it. For example, in GUTs the  $\mu$ -puzzle is only an extension of the celebrated doublet-triplet problem,<sup>18</sup> *i.e.*, the splitting of the  $\mathbf{5}$  and  $\mathbf{\bar{5}}$  of  $SU(5)$  to a light pair of  $SU(2)$  doublets

and a heavy pair of color  $SU(3)$  triplets. Assuming that such splitting occurs with exactly massless doublets in the limit of global supersymmetry, once the  $\mathcal{O}(m_{3/2})$  SSB terms for the heavy GUT fields are introduced, they explicitly break the global supersymmetry and they can appropriately shift the doublet mass, *i.e.*, the  $\mu$ -parameter.<sup>19</sup>  $X$  in this case is either a heavy singlet or a field in the adjoint representation of the GUT group. A different and more recent proposal,<sup>21</sup> about which we will elaborate below, relates the  $\mathcal{O}(m_{3/2})$  shift in  $X$  and  $F_X$  to a radiative generation of a tadpole term for  $X$  once supersymmetry is broken in the hidden sector.  $X$  in this case is a (total) singlet which decouples at a scale  $\sim m_{3/2}\Lambda_P$  but its VEVs are slightly shifted from zero  $X \sim m_{3/2} + \theta^2 m_{3/2}^2$ .

An alternative to renormalizable operators (and an option which is common in model building) is that the symmetries in the ultra-violet theory allow only for non-renormalizable operator realization of  $\mu$ . The leading  $n = 2$  operators require  $X \simeq m_{3/2}\Lambda_P$ , the geometrical mean of the weak and Planck scale which also corresponds (in the supergravity framework) to the scale of supersymmetry breaking in the hidden sector,  $\Lambda_{\text{SUSY}}$ . This scale also corresponds to the invisible axion window and suggests that an anomalous symmetry such as a Peccei-Quinn<sup>3,14</sup> or a  $R$ -symmetry<sup>12,13</sup> may dictate, in this case, the selection rules for the operators.  $X$  is related to the breaking of such a symmetry, though it could be in many cases a hidden sector field (since its coupling is suppressed by powers of  $\Lambda_P$ ). It is possible to relate the symmetry breaking (particularly in the case of a  $R$ -symmetry) to supersymmetry breaking in the hidden sector. The SSB mixing parameters could arise from terms in the second ( $D$ -term) integral in Eq. (10) with fields  $X$  (and  $Z$ ) corresponding to hidden sector fields with  $F \sim m_{3/2}\Lambda_P$ . The  $n = 3$  case is also quite interesting, since one could identify  $X^3 \equiv W$  where (assuming cancellation of the cosmological constant) the superpotential VEV is given by that of the hidden superpotential<sup>22</sup>  $\langle W_{\text{hidden}} \rangle = m_{3/2}\Lambda_P^2$ , which parameterizes in this case supersymmetry breaking. Alternatively,  $X^3 \equiv W^\alpha W_\alpha$  could be identified with a (hidden sector) gaugino condensate as long as  $\langle W_{\text{hidden}}^\alpha W_{\alpha \text{ hidden}} \rangle = m_{3/2}\Lambda_P^2$  as occurs in non-renormalizable hidden sector models.<sup>8</sup> ( $W_\alpha$  is the chiral representation of the gauge superfield.)

The last possibility suggests that the hidden-visible gravity-mediated mixing occurs in the holomorphic gauge kinetic function  $f_{\alpha\beta} = \delta_{\alpha\beta}(1 + (H_U H_D/M^2) + \dots)/2g^2$  (which determines the gauge Lagrangian  $f_{\alpha\beta} W_\alpha W_\beta$  and  $g$  here is the relevant gauge coupling) rather than in the superpotential. (Note that the holomorphicity of the Higgs bilinear is explicitly exploited.) Such a general form of the gauge kinetic function can also lead to a radiative but quadratically divergent (and thus, not negligible) contribution to  $\mu$  which

Table 1. The required field values for consistent generation of  $\mu$  and possible symmetry sources for selection rules. In supergravity mediation  $\Lambda_{\text{SSB}} \simeq m_{3/2}$ .

$n$	$\langle X \rangle$	Symmetry
1	$\Lambda_{\text{SSB}}/a$	$Z_3$ , gauged $U(1)$ , GUT
2	$M\Lambda_{\text{SSB}}/a$	Peccei-Quinn, $R$
3	$M^2\Lambda_{\text{SSB}}/a$	$R$

is proportional to the SSB gaugino mass.<sup>23</sup> The mixing could also occur in the non-holomorphic Kähler potential, where again, one can exploit the holomorphicity of the Higgs bilinear and write<sup>24</sup> the terms which appear in the second ( $D$ -term) integral in Eq. (9).  $X$  is a hidden sector field and once it is integrated out  $\int d^2\bar{\theta} X^\dagger \rightarrow F_X^\dagger \simeq m_{3/2}\Lambda_P$  then this term is reduced to a usual superpotential term. (Note that similar  $X^\dagger QQ^\dagger$  must be forbidden to avoid flavor non-diagonal sfermion masses, as could be done by a  $R$  symmetry.) In fact, in supergravity (rather than the rigid supersymmetry of the effective theory which is implicitly assume in writing Eqs. (9) and (10)) it is sufficient to write a Kähler potential which includes the holomorphic term  $K = H_U H_D + \text{H.c.}$ . Using a Kähler transformation

$$K \rightarrow K - (H_U H_D + \text{H.c.}) \text{ and } W \rightarrow e^{(H_U H_D / \Lambda_P^2)} W,$$

and substituting  $W = \langle W_{\text{hidden}} \rangle + W_{\text{MSSM}}$  where  $\langle W_{\text{hidden}} \rangle \simeq m_{3/2}\Lambda_P^2$  and  $W_{\text{MSSM}}$  is given by Eq. (1) but without the  $\mu$ -term, reproduces the same result. Hence, one can identify this operator with the first  $n = 3$  operator discussed above. (Note, however, that the renormalizable and non-renormalizable Kähler potential operators have a different holomorphy structure in this case.)

The identity  $m_{3/2} \simeq \Lambda_{\text{SSB}}$  and the multitude of available tree-level operators more than resolves the  $\mu$  problem in supergravity and explains the correlation  $\mu \simeq \Lambda_{\text{SSB}}$ . Some of the results are summarized in Table 1. Note that though one can clearly understand  $\mu \sim \Lambda_{\text{SSB}} \sim M_{\text{gluino}}$ , this relation is not renormalization group invariant. The fine tuning issue can then be

phrased as understanding the special value  $\mu/M_{\text{gluino}}$  at the weak rather than the ultra-violet scale. The situation improves in this respect in models with lower mediation scales, but as we shall see, different complications arise in these cases.

It is possible that the sparticle spectrum contains both multi-TeV sfermions (superpartners of the first and second family sfermions) and sub-TeV Higgs particles, gauginos, and third family sfermions. The former ensure decoupling of potential contributions to sensitive flavor changing neutral currents observables, while the latter allow for natural solutions to EWSB.<sup>25</sup> If the gravitino mass is still in the sub-TeV<sup>26</sup> then  $\mu \sim m_{3/2}$  is sufficient. However, if the gravitino sets the scale for the heavy sector,<sup>27</sup> one must invoke a Peccei-Quinn (or other) symmetry (alongside the  $R$ -symmetry whose role is now to suppress the gaugino masses) in order to ensure sub-TeV Higgs mass parameters.

An alternative to gravity mediation is the gauge mediation framework in which the mass-dimension one and two SSB parameters are induced by gauge loops at one and two loop orders, respectively, so that  $m_{\text{gaugino}} \sim m_{\text{sfermion}} \sim \Lambda_{\text{SSB}} \sim (\alpha/4\pi)\Lambda_{\text{M}} \gg m_{3/2}$ . Here,  $\alpha$  is a properly renormalized SM gauge coupling and  $\Lambda_{\text{M}} \sim (4\pi/\alpha)\Lambda_{\text{SSB}} \sim (4\pi/\alpha)(4\pi/h_t)\Lambda_{\text{W}} \sim 10^{4-6}$  GeV is a messenger scale, *i.e.*, the mass scale of the messenger fields which communicate between the SM gauge interactions and a low energy supersymmetry breaking sector (the equivalent of the supergravity hidden sector). While supergravity interactions cannot be eliminated, their effects are typically suppressed by the small gravitino mass  $m_{3/2} \sim \Lambda_{\text{M}}^2/\Lambda_{\text{P}}$ . (For an exception, see Ref. 28.) However, the mixing terms  $\mu H_U H_D$  and  $m_{H_U H_D}^2 H_U H_D$  explicitly break the Peccei-Quinn symmetry and therefore cannot arise from gauge loops. Instead, they could arise from Yukawa loops if the messenger sector is extended appropriately. (In fact, if  $\mu \neq 0$  then  $m_{H_U H_D}^2 \sim \mu \Lambda_{\text{SSB}}$  arises at two-loop order from one-loop (gauge) renormalization once the gaugino mass is induced at one loop,<sup>29</sup> but this is immaterial for our discussion here.) However, in the case of Yukawa interactions one does not find a hierarchy similar to  $m_{\text{sfermion}}^2/m_{\text{gaugino}} \sim [(\alpha/4\pi)^2 \Lambda_{\text{M}}^2]/[(\alpha/4\pi)\Lambda_{\text{M}}] \sim \Lambda_{\text{SSB}}$  which appears in the case of gauge loops. Instead, one typically finds<sup>30</sup>  $m_{H_U H_D}^2/\mu \sim [(h/16\pi^2)\Lambda_{\text{M}}^2]/[(h/16\pi^2)\Lambda_{\text{M}}] \sim \Lambda_{\text{M}}$ . That is, both dimension one and two mixing parameters arise at the same loop order.

The operator language is convenient for comparison with the supergravity case. The  $\mu$  parameter would arise from Eq. (9) with  $X \sim \sqrt{|F_X|} \sim \Lambda_{\text{M}}$  given by the messenger scale and (assuming one-loop effects)  $a \sim a' \sim 1/16\pi^2$ . Similarly, the SSB  $m_{H_U H_D}^2$  parameter arises from Eq. (10) with  $b \sim b' \sim 1/16\pi^2$ . (Some coefficients may vanish, depending on the exact structure one as-

sumes.) While in the case of tree-level supergravity operators one has  $m_{H_{UD}}^2 \sim \mu \Lambda_{\text{SSB}} \sim \mu^2$ , if both arise quantum mechanically at the same loop order then one has instead  $m_{H_{UD}}^2 \sim \mu \Lambda_{\text{M}}$ . As a result, a new hierarchy problem that shadows the gauge mediation framework emerges. One avenue to resolve the gauge-mediation variant of the  $\mu$  puzzle is to allow for  $\mu$  and  $m_{H_{UD}}^2$  generation at tree-level and hence to reduce the problem to its “supergravity form”, or alternatively to allow for  $m_{H_{UD}}^2$  generation only at higher loop level. Both approaches require the introduction of dedicated singlets  $X$  and of non-trivial structures and/or interactions. A NMSSM realization of the former will be presented below.

It was recently proposed<sup>31</sup> that supergravity mediation may also arise only at the quantum level with  $\Lambda_{\text{SSB}} \sim (\alpha/4\pi)m_{3/2}$  and the gravitino mass  $m_{3/2} \gtrsim 10$  TeV, which is similar in size to the messenger scale of gauge mediation. This is the anomaly mediation mechanism. There, the theory is assumed to preserve at all orders a geometrical separation of the hidden and observable sectors, and supersymmetry breaking effects in the observable sector can arise only through the interactions of the supergravity multiplet. This leads to relations similar to those of gauge mediation but with quite different and distinct coefficients. Specifically, the mediation of supersymmetry breaking due to the supergravity multiplet can be extracted (in an appropriate gauge) by introducing appropriate powers of a background field  $\phi = 1 + \theta^2 m_{3/2}$ , the holomorphic compensator, to the different operators (so that the action is rendered Weyl invariant). The compensator, which in this parameterization is the only source of supersymmetry breaking in the observable sector, then allows for  $\mu$  and  $m_{H_{UD}}^2$  generation from the  $D$ -term integral in Eq. (9). However, the tree level operator leads in this case to  $\mu \sim m_{3/2} \gg \Lambda_{\text{W}}$ . Even if one sets  $a' \ll 1$ , the hierarchy problem described in the case of gauge mediation appears. Thus, one must forbid tree-level generation and rely on radiative generation (except, in principle, in the case of the NMSSM with  $\langle X \rangle \sim \Lambda_{\text{SSB}}$ ). Proposed solutions<sup>32</sup> have to rely on extended structures and, furthermore, disturb the minimality which is the essence of the initial proposals of this framework.

Lastly, we would like to point out a higher order  $D$ -term operator,<sup>33</sup>

$$\text{SSB Higgsino mixing } (\tilde{\mu}) : c \int d^2\theta d^2\bar{\theta} D H_1 D H_2 \left[ \frac{X X^\dagger}{M^3} \right] + \text{H.c.}, \quad (11)$$

where  $D = \partial/\partial\theta$  is the covariant superspace derivative with mass dimension  $[D] = 1/2$ . Its operation selects the Higgsino components of the Higgs superfields  $H_{D,U}$  while the superspace integration leads to  $\tilde{\mu} \sim |F_X|^2/M^3$ . Such a SSB Higgsino mass leads to the issue of “non-standard” supersym-

metry breaking terms<sup>34</sup> as it can always be rotated into a combination of the usual  $\mu$ -term and non-holomorphic  $A'H^\dagger\phi_L\phi_R$  supersymmetry breaking terms ( $\phi_{L,R}$  are “left and right-handed” sfermions). As long as the low-energy theory does not contain a singlet which couples to light fields, such terms do not destabilize the hierarchy. However, their realization may require  $\sqrt{F_X} \sim M \ll \Lambda_P$ , *i.e.*, a truly non-standard realization. Other non-standard low-energy scenarios were also proposed.<sup>35</sup>

## 4 Recent Models

Equipped with the above “catalog” of operators and their manifestations, we proceed to present two specific realizations in some more detail. We choose two very different realization of the  $n = 1$  tree-level NMSSM operator, the first in the context of gravity mediation while the second in the context of gauge mediation.

### 4.1 Stabilized singlets and supergravity

In Ref. 21 it was proposed that while a singlet field with a VEV  $\langle X \rangle \sim \sqrt{F_X} \sim m_{3/2}$  may induce  $\mu$  and  $m_{H_{UD}}^2$  in a supergravity scenario with a (hidden-sector) spontaneous supersymmetry breaking scale  $\Lambda_{\text{SUSY}} \sim 10^{11}$  GeV, its mass may be  $m_X \sim \mathcal{O}(\Lambda_{\text{SUSY}})$ . This proposal combines aspect of schemes based non-renormalizable supergravity operators with some of the basic features of the NMSSM, and we will examine it in some detail.

As noted above, the NMSSM has two well-known problems. At the renormalizable level, the NMSSM has a  $Z_3$  symmetry. If that symmetry is preserved to all orders, then the VEV of  $X$  will break the symmetry at the weak scale and produce cosmologically dangerous domain walls. If, on the other hand, the  $Z_3$  symmetry is not preserved by higher-order terms in the Lagrangian, then  $X$  carries no conserved quantum numbers. In this latter case,  $X$  will generically develop tadpoles, in the presence of spontaneously-broken supersymmetry, whose quadratic divergences are cut off by the Planck scale<sup>36,33</sup>. The resulting shift in the potential for  $X$  causes it to slide to large values far above the weak scale. If it were to couple to the MSSM Higgs fields, they would receive unacceptably large masses, destabilizing the weak scale. Therefore one concludes that not only do singlets fail to provide a viable  $\mu$ -parameter, but they cannot even be allowed to couple to light fields.

It is possible, however to solve this destabilization problem of the NMSSM, while at the same time introducing a new visible sector interaction whose scale will naturally fall at  $\Lambda_{\text{SUSY}}$ . Singlets can now couple to MSSM fields, and in

particular, can provide a dynamical  $\mu$ -term at the weak scale. The model presented here demonstrates a very general mechanism, and it already contains all of the ingredients necessary to be phenomenologically viable (though these aspects will not be developed here).

Consider a superpotential

$$W = \lambda_H X H \overline{H} + \lambda_\Sigma X \Sigma \overline{\Sigma} \quad (12)$$

where  $H, \overline{H}$  carry charges  $\pm 1$  under a gauge symmetry  $U(1)_H$ ,  $\Sigma, \overline{\Sigma}$  are charged  $\pm 1$  under another gauge symmetry  $U(1)_\Sigma$ , and  $X$  is a gauge singlet. We require  $\Sigma, \overline{\Sigma}$  to be neutral under  $U(1)_H$  and, for simplicity, assume that  $H, \overline{H}$  are also neutral under  $U(1)_\Sigma$ , though they need not be. The gauge  $D$ -term for  $U(1)_\Sigma$  is then simply

$$D_\Sigma = g_\Sigma (|\Sigma|^2 - |\overline{\Sigma}|^2), \quad (13)$$

and similarly for  $D_H$ . To apply this toy model to the MSSM, we identify  $H, \overline{H}$  as the usual Higgs doublets, and extend  $U(1)_H$  to the Standard Model gauge group;  $\Sigma$  and  $\overline{\Sigma}$  are new fields charged under a new gauge symmetry  $U(1)_\Sigma$ .

At the level of the superpotential, there exists the usual  $Z_3$  which forbids explicit mass terms from appearing in  $W$ <sup>15</sup>. This symmetry is broken by  $X \neq 0$ , which could lead to creation of electroweak scale domain walls (via the Kibble mechanism). The appearance of an  $X^3$  term is forbidden by an  $R$ -symmetry under which  $R(W) = 2$  and  $R(X) = 0$ . However, we will assume that the  $Z_3$  symmetry of the superpotential is only an accidental symmetry. This is a natural expectation since global symmetries are generally not preserved by quantum gravity effects (unless they are remnants of broken gauge symmetries). In particular, we expect that gravity-induced global symmetry breaking will appear as non-renormalizable, explicit symmetry-breaking terms in the Kähler potential. The  $Z_3$  symmetry can thus be a symmetry of the effective superpotential without being a symmetry of the entire action. This is equivalent to the statement that the  $X$  field is a true singlet, carrying no conserved quantum numbers.

After supersymmetry breaking, non-zero tadpoles for  $X$  will generically arise with light chiral fields circulating in the loops<sup>36,33,37,38,28</sup>. These tadpoles appear quantum mechanically due to supergravity corrections from Planck-suppressed operators. Because the exact source of the couplings which generate the tadpoles is highly model-dependent, we do not know *a priori* at what loop order non-zero contributions are generated. For example, it is known that for a flat Kähler metric, non-zero tadpoles do not arise until two-loops<sup>37,38</sup>; however, for a non-flat metric they may arise at one-loop.

The contribution to the effective (component field) potential coming from the tadpole can be parameterized as

$$V_{\text{linear}} \sim \beta \epsilon m_{3/2} \Lambda_P F_X + \gamma m_{3/2}^2 \Lambda_P X + \text{H.c.} \quad (14)$$

where  $\epsilon$  is the maximum of a set of measures  $\epsilon_i$  of the supersymmetry breaking field VEVs (in Planckian units), and  $\gamma, \beta$  are complex coefficients which include the loop suppression factors  $(16\pi^2)^{-n}$  (typically  $n = 1$  or  $n = 2$ ) as well as counting factors  $N$  which sum all unknown coefficients, and so whose magnitudes are roughly  $\mathcal{O}(10^{-4} - 1)$ . (We implicitly assume that  $N$  is such that the calculation remains perturbative, *i.e.*,  $N \lesssim 100$ .)

Combining Eqs. (14) and (2) one can write down the full scalar potential after supersymmetry breaking, including supergravity-mediated soft masses as well as the tadpole contributions. Begin by considering the contributions to the scalar potential involving the  $F_X$  auxiliary field:

$$V_{F_X} = (\beta \epsilon m_{3/2} \Lambda_P F_X + \text{H.c.}) - |F_X|^2 - \left( F_X \frac{\partial W}{\partial X} + \text{H.c.} \right) \quad (15)$$

where the first term is the contribution of the tadpole. On integrating out all auxiliary fields, one finds that  $F_X$  is shifted from its canonical form by the tadpole contribution:

$$F_X^\dagger = -\frac{\partial W}{\partial X} + \beta \epsilon m_{3/2} \Lambda_P, \quad (16)$$

while all other  $F$ -terms (*e.g.*  $F_\Sigma$  and  $F_H$ ) are canonical. The  $D$ -terms associated with the gauge fields also take their canonical forms.

The full scalar potential after soft supersymmetry breaking can then be written:

$$\begin{aligned} V = & \sum_i m_i^2 |\varphi_i|^2 + |\lambda_\Sigma X|^2 (|\Sigma|^2 + |\bar{\Sigma}|^2) + |\lambda_H X|^2 (|H|^2 + |\bar{H}|^2) \\ & + m_{3/2}^2 \Lambda_P (\gamma X + \gamma^\dagger X^\dagger) + |\lambda_\Sigma \Sigma \bar{\Sigma} + \lambda_H H \bar{H} - \beta \epsilon m_{3/2} \Lambda_P|^2 \\ & + \frac{g_\Sigma^2}{2} (|\Sigma|^2 - |\bar{\Sigma}|^2)^2 + \frac{g_H^2}{2} (|H|^2 - |\bar{H}|^2)^2, \end{aligned} \quad (17)$$

where the first term represents the gravitationally-induced soft supersymmetry breaking masses,  $m_i^2 \sim m_{3/2}^2$ , for the fields  $\varphi_i = \{X, \Sigma, \bar{\Sigma}, H, \bar{H}\}$ , and the superpotential derivative in Eq. (2) was replaced with the right hand side of Eq. (16). For simplicity, we ignore hereafter holomorphic trilinear ( $A$ ) and bilinear ( $B$ ) SSB terms; they do not change our results substantially. Note that the potential as written requires that  $m_X^2 \geq 0$  in order to be bounded from below (this condition is modified in the presence of  $B$ -terms). Indeed,



one expects  $m_X^2 > 0$  at tree level and it will only be driven negative if its coupling to either of the two sets of Higgs fields is fairly large. Henceforth we will take all soft squared-masses to be equal to  $m_{3/2}^2 > 0$ .

To continue further, we take  $\epsilon \simeq 1$  which is the generic choice; small deviations of  $\epsilon$  away from 1 can be absorbed into  $\beta$ . Writing down the minimization conditions for the potential is straightforward, but as the potential is quite complicated, it has many local minima besides the true global one. However, there are two lowest-lying minima, both along directions that are  $D$ -flat up to weak-scale corrections, *i.e.*,  $\Sigma \simeq \overline{\Sigma}$  and  $H \simeq \overline{H}$ .

At a first minimum, denoted  $V_1$ ,

$$\begin{aligned}\Sigma = \overline{\Sigma} = H = \overline{H} &= 0, \\ X &\simeq -\gamma^\dagger \Lambda_P, \\ |F_X| &\simeq |\beta m_{3/2} \Lambda_P|, \\ V_1 \equiv V_{\min} &\simeq (|\beta|^2 - |\gamma|^2) m_{3/2}^2 \Lambda_P^2.\end{aligned}$$

This minimum represents the case usually considered in the literature for singlets with non-zero tadpoles — their VEVs are pulled up to the Planck scale, taking with them any matter to which they couple. This is precisely the reason it was argued<sup>33,38</sup> that the VEV of a true singlet cannot be responsible for the  $\mu$ -term in the MSSM.

At a second minimum,  $V_2$ ,

$$\begin{aligned}\Sigma \overline{\Sigma} &= \frac{\beta m_{3/2} \Lambda_P}{\lambda_\Sigma}, \\ H = \overline{H} &= 0, \\ X &\simeq -\frac{\gamma^\dagger}{2|\lambda_\Sigma \beta|} m_{3/2}, \\ F_X &\sim m_{3/2}^2, \\ F_{\Sigma, \overline{\Sigma}} &\simeq \lambda_\Sigma m_{3/2}^3 \Lambda_P^{1/2}, \\ V_2 \equiv V_{\min} &\simeq \frac{1}{|\lambda_\Sigma|} \left( |\beta| - \left| \frac{\gamma^2}{2\beta} \right| \right) m_{3/2}^3 \Lambda_P.\end{aligned}$$

The  $\Sigma$ -fields receive VEVs of  $\sim \sqrt{m_{3/2} \Lambda_P}$  to cancel off the  $F_X$  contribution to the potential. These large  $\Sigma$ -VEVs then produce masses for the  $X$ -field (through the  $F_\Sigma$  terms) which stabilizes the  $X$ -VEV against the tadpole-induced linear potential. The resulting VEV of  $X$  is then only  $\langle X \rangle \sim m_{3/2} \simeq \Lambda_W$ ! Any gauge symmetry carried by the  $\Sigma$ -fields will be broken at the scale of their VEVs. Up to the loop factors buried in  $\beta$ , this is the intermediate

scale,  $\Lambda_{\text{SUSY}}$ . In fact, one may interpret the physics at this minimum as the tadpoles communicating to the  $\Sigma$ -fields the true scale of supersymmetry breaking, up to the loop factors.

There is also a third minimum,  $V_3$ , which is identical to  $V_2$  except that the would-be MSSM Higgs fields,  $H$  and  $\overline{H}$ , play the role of  $\Sigma$  and  $\overline{\Sigma}$  and receive VEVs  $\sim \Lambda_{\text{SUSY}}$ , with  $\lambda_H$  replacing  $\lambda_\Sigma$  in all expressions. This is clearly *not* the desired minimum but is instead another example of how the tadpole can destabilize the weak scale. (Note that points at which  $H, \overline{H}, \Sigma, \overline{\Sigma}$  all get VEVs simultaneously are not even local minima of the potential.)

By observation, for  $|\beta|^2 > |\gamma|^2$  and  $|\lambda_H| < |\lambda_\Sigma|$   $V_2$  corresponds to the global minimum and  $X$  develops a VEV of the order of magnitude of the weak scale and provides a  $\mu$ -term of the correct size for light MSSM Higgs fields. Note that simple inequality is all that is needed to ensure that a gauge hierarchy develops; no large hierarchy is needed between the two couplings themselves. Unequal soft masses shift the condition slightly, but the same basic result will always hold. Thus we conclude that the  $X$ -VEV can in fact provide the  $\mu$ -term of the MSSM as long as  $|\lambda_H| < |\lambda_\Sigma|$ . The whole question of which gauge group is broken at the scale  $m_{3/2}$  and which at  $\Lambda_{\text{SUSY}}$  may rest entirely on the relative size of two couplings ( $\lambda_H$  and  $\lambda_\Sigma$ ) whose ratio is generically  $\mathcal{O}(1)$ !

It is well-known that models with a dynamical  $\mu$ -term can contain a Peccei-Quinn ( $PQ$ ) symmetry which would be spontaneously broken and thus create an unwanted axion at the weak scale. To examine this possibility, promote the  $PQ$ -symmetry to the previously discussed  $R$ -symmetry under which  $X$  is neutral and all other superfields are singly charged. However,  $R(\beta m_{3/2} \Lambda_P) = 2$  explicitly breaks the symmetry and the would-be axions are all given masses near the intermediate scale, rendering them harmless. But there still remains a residual  $PQ$  symmetry in the MSSM Lagrangian. This too is explicitly broken, this time by  $F_X \sim \Lambda_W^2$ , which generates a  $m_{H_{UD}}^2$ -term in the Higgs sector ( $\sim F_X^\dagger H \overline{H}$ ) and gives mass to the pseudoscalar Higgs/would-be axion.

In conclusion, a new mechanism for obtaining a weak-scale  $\mu$ -parameter by adding a total singlet in conjunction with a new gauge interaction and its accompanying Higgs sector was presented. We have used the tadpoles endemic to models with singlets to drive the breaking of the new symmetry at the intermediate scale  $\sqrt{\Lambda_W \Lambda_P}$ , to remove all vestiges of the singlet from the low-energy theory, and to render all would-be axions harmless. The intermediate scale may be further associated with the scale of a symmetry breaking governing the mass scale right-handed neutrino<sup>39</sup>, for example. Such models may be described as a “decoupled NMSSM” in which the NMSSM singlet is

integrated out near the scale of supersymmetry breaking but its traces remain in the low-energy theory. Further details on the model, its vacuum structure (at tree and one-loop levels), and details and possible interpretation of the corresponding intermediate-scale physics can be found in Ref. 21.

#### 4.2 Gauge non-singlets and gauge mediation

Next, we turn to consider an explicit realization of the NMSSM within the framework of gauge mediation. Given the special form of the  $\mu$ -puzzle,  $m_{H_{UD}}^2 = \mu\Lambda_M \sim \mu\Lambda_{\text{SUSY}} \gg \Lambda_W^2$ , there are two possible classes of solutions. Either  $\mu$  and  $m_{H_{UD}}^2$  both arise at tree level so that their size is not determined by a (Yukawa) loop suppression factor raised to some power, or alternatively, they arise at different loop orders and hence with different powers of the suppression factor. Realizations of these ideas, however, are far from straightforward.

The most successful attempts to address this new hierarchy problem fall along these lines and involve in one fashion or another the details of the high-energy (supergravity) theory, and in that sense they are high-energy solutions. For example, one can invoke<sup>30</sup> a radiative linear term generated by messenger-scale singlet interactions. The linear term shifts a singlet field  $X$  (which interacts with the Higgs doublets) to a scale which is suppressed by a loop factor in comparison to the messenger scale. The shifts in the scalar and auxiliary components of  $X$ , which induce  $\mu$  and  $m_{H_{UD}}^2$ , respectively, arise at different loop orders, evading the above described hierarchy problem. The superpotential (or equivalently – the Kähler potential) couplings must be fixed by the high-energy ( $Q > \Lambda_M$ ) theory. In particular, a scale associated with a tree-level linear term must be fixed to be  $\mathcal{O}(\Lambda_M)$ . Alternatively, it was pointed out<sup>28</sup> that a radiative linear term in a singlet field  $X$  is typically generated by supergravity and is suppressed by only one inverse power of the Planck mass  $M_P$ , as demonstrated in Sec. 4.1 above. Hence, it can still play an important role in the low-energy theory. It shifts the singlet field  $X \sim (\Lambda_{\text{SUSY}}^4/\kappa^2 M_P)^{1/3}$  (assuming in this case  $W(X) \sim (\kappa/3)X^3$  and  $\epsilon \rightarrow 0$ ). The singlet Yukawa interaction with the Higgs doublets then generates the desired parameters at tree-level  $\mu^2 \sim m_{H_{UD}}^2 \sim X^2 \sim \Lambda_W^2$  (assuming that supersymmetry is spontaneously broken at a scale  $\Lambda_{\text{SUSY}} \sim (4\pi/\alpha)\Lambda_M \sim \mathcal{O}(10^{6\pm 1})$  GeV). In this case no new scales are introduced by hand, but there is still dependence on the high-energy theory. (A somewhat similar application of supergravity to the problem was proposed in Ref. 40.)

Both proposals qualify as versions of a “decoupled NMSSM” in the sense described in Sec. 4.1 above, only that the decoupling scale is now  $\Lambda_M$ . Here,

however, we point out a distinctive possibility that the singlet field is not a gauge singlet but only a SM singlet  $S$  which does not decouple at the messenger scale (and hence will be denoted by  $S$ ), *i.e.*, a gauged NMSSM. This possibility was discussed in Ref. 41, which we follow. Specifically, let us assume the extension  $(S)SM \rightarrow (S)SM \times U(1)'$ , and that  $S$  carries a charge  $Q_S = -(Q_{H_1} + Q_{H_2})$  under the additional Abelian symmetry so that a Yukawa term  $W \sim h_s S H_1 H_2$  is allowed. In turn, a scale  $\Lambda' \sim \langle S \rangle \lesssim \Lambda_M$ , which is associated with the breaking of the  $U(1)'$ , must be introduced, or preferably, induced. The  $\mu$  and  $m_{H_{UD}}^2$  parameters are induced by the singlet interactions at tree-level and the various  $\mu$  problems of gauge mediation are solved in this case by the low-energy dynamics associated with this new scale.

The scale  $\Lambda'$  could be generated radiatively and is a function in this case of  $\Lambda_M$  and of  $\mathcal{O}(1)$  Yukawa couplings. A coupling between  $S$  and exotic quarks, e.g.,  $D$  and  $D^c$  singlets with hypercharge  $\pm(1/3)$ , generates negative corrections to the SSB parameter  $m_S^2$  so that  $m_S^2(\Lambda') < 0$  and  $S$  acquires a VEV. This is essentially a  $U(1)'$  version of the well-known radiative symmetry breaking (RSB) mechanism that is responsible in the MSSM for the generation of the negative mass term in the SM Higgs potential and the satisfaction of the conditions for EWSB discussed in Sec. 2. A similar idea<sup>17</sup> was mentioned above in the context of supergravity and high-energy (gravity) mediation of supersymmetry breaking. In that case, like RSB in those models, the large evolution interval enables one to render  $m_S^2 < 0$  somewhere above the weak scale. In the supergravity case the superpotential interactions generate  $|\mu| \sim h_s \langle S \rangle$  while trilinear SSB terms  $V_{SSB} \sim \dots + h_s A_s S H_1 H_2 + \text{H.c.} + \dots$  generate  $m_{H_{UD}}^2 = A_s h_s \langle S \rangle$ . Since all parameters in the gravity-mediation framework are of the same order of magnitude as the gravitino mass (which is fixed in that case  $m_{3/2} \sim \Lambda_W$ ), then  $h_s \langle S \rangle$  is expected to be of the same order of magnitude as well. This leads to a successful solution to the  $\mu$ -problem in high-energy supergravity models. In contrast to the supergravity framework, in gauge mediation the evolution interval is short; in addition, trilinear parameters are highly suppressed  $A \sim (\alpha/4\pi)^2 \Lambda_M \ln \Lambda_M$  so that  $m_{H_{UD}}^2 \propto A\mu$  is also suppressed, even if  $\mu$  is acceptable. (Formally,  $m_{H_{UD}}^2$  now arises at a too high loop order!) While the small  $A$  parameters remain a constraint, the shorter evolution interval is more than compensated (as for the case of RSB in these models) by the large hierarchy within the SSB parameters  $m_D^2/m_H^2/m_S^2 \sim \alpha_3^2/\alpha_2^2/\alpha_{1'}^2$  (where  $m_D^2$ ,  $m_H^2$  and  $m_S^2$  are the soft mass-squares of the exotic quark  $D$ , Higgs doublet  $H$  and singlet  $S$  and  $\alpha_{3,2,1,1'}$  are the  $SU(3)$ ,  $SU(2)$ ,  $U(1)$ , and  $U(1)'$  gauge couplings). In fact, the messengers may not transform under  $U(1)'$ , in which case  $m_S^2(\Lambda_M) = 0$ . For  $\alpha_{1'} = \mathcal{O}(\alpha_Y)$ , which we will assume, the exact boundary condition for  $m_S^2$  does not affect

our discussion and for simplicity we assume hereafter that the messengers are indeed invariant under  $U(1)'$ . (It can affect, however, the singlet slepton spectrum, which is otherwise given in gauge mediation only by hypercharge loops.)

A radiatively induced  $\langle S \rangle$  as a source of  $\mu$  in the case of a gauge singlet  $S$  was considered previously in the context of gauge mediation.<sup>42</sup> It was found that the singlet must couple to exotic quarks with large Yukawa couplings, as naturally occurs in the context of  $U(1)'$ . In the gauge singlet case, however, the superpotential must contain a  $S^3$  term so that the potential contains quartic terms  $V \sim |\partial(S^3 + SH_1H_2)/\partial S|^2$  which stabilize it. Like the gravity-mediation versions of the NMSSM those models suffer from the problem of a spontaneously broken global  $Z_3$  symmetry (under which  $S^3$  is invariant) which results in unacceptable domain walls at a low-energy epoch. In the gauged case  $S$  is not a singlet and  $S^3$  terms are not gauge invariant and are automatically forbidden. Instead, the potential is stabilized by  $U(1)'$  gauge  $D$ -terms  $V \sim \dots + (g_1^2/2)(Q_S|S|^2 + Q_{H_1}|H_1|^2 + Q_{H_2}|H_2|^2)^2 + \dots$  (which are not available for a gauge singlet  $S$ ). The  $Z_3$  symmetry is now only a (harmless) subgroup of the gauged  $U(1)'$ . While in the non-gauged case the former source of the quartic terms also generates an additional contribution to  $m_{H_{UD}}^2 \sim S^2$ , this is not possible in the gauged case (with only one singlet).

In either the gauged or non-gauged case, the potential also exhibits an approximate phase ( $R$ ) symmetry, which exists in models with only Yukawa superpotential terms and corresponds to a rotation of all fields by the same phase. It is broken spontaneously by  $\langle S \rangle$  and explicitly by tri-linear  $A$ -terms. The explicit breaking is, however, suppressed by the smallness of the  $A$ -parameters. Nevertheless, we will find below that in spite of the suppressed  $A$ -parameters it is possible to generate  $m_{H_{UD}}^2$  and break the phase symmetry strongly enough to avoid the light pseudo Goldstone boson which otherwise appears. Specifically, as will be shown below, it is very likely that in the  $U(1)'$  scenario  $\Lambda_W \ll \langle S \rangle \lesssim \Lambda_M$  and hence,  $m_{H_{UD}}^2 \sim h_s A_s \langle S \rangle \sim A_s \mu$  is a geometric mean of a small parameter and a large VEV. It implies a somewhat large value of  $|\mu| \sim \mathcal{O}(1 \text{ TeV})$ . However, this typically occurs in gauge mediation as a result of RSB constraints in the presence of a heavy gluino. Alternatively, in models with two singlets a superpotential term  $SS'^2$  could be gauge invariant, and  $\langle S' \rangle \sim \langle S \rangle$  could generate an additional contribution to  $m_{H_{UD}}^2 \sim \langle S' \rangle^2$ , just as in the non-gauged case. (Note that in the non-gauged case the  $U(1)'$  rotations – explicitly broken by the  $S^3$  terms – correspond to global transformations and there is one additional pseudo Goldstone boson.) Here we confine ourselves to models with only one SM singlet  $S$ .

It is particularly interesting to note that in the models with only one SM

singlet there appear only two new phases which can be rotated away, and hence there are no new physical phases. This is because there is only one common phase to all gaugino mass and the radiatively-induced  $A$  parameters, while the phase of  $m_{H_{UD}}^2$  is given in this case by the phases of  $\mu$  and  $A$ . Hence, after  $R$  and Peccei-Quinn rotations no physical phases appear in the soft parameters. This eliminates new contributions to CP violating amplitudes such as the electron dipole moment, which are flavor conserving and which generically appear at unacceptable levels even in gauge-mediation models.<sup>43</sup>

The stabilization due to the  $D$ -terms and the generation of the  $A$  terms then open the door to new (low-energy) solutions to the  $\mu$ -problem in gauge mediation. The mechanism is quite different from that of the non-gauged case since the quartic coupling is given, in principle, by a fixed gauge coupling rather than by a free superpotential coupling;  $m_{H_{UD}}^2$  must depend on overcoming the suppression of the tri-linear couplings  $A$ ; and the scale  $\langle S \rangle$  is a physical scale with observable consequences. Hence, it corresponds to a distinctive and interesting option. The  $U(1)'$  models predict, in addition to the extra matter and the associated rich spectrum, an extra gauge boson,  $Z'$ . The corresponding phenomenology is similar to that of any other model with  $Z'$ , except that  $m_{Z'}^2 \sim -(Q_S/2)m_S^2$  is large, given that  $|m_S^2|$  is controlled by the large exotic quark SSB parameters. Typically we find  $m_{Z'} \simeq \mathcal{O}(1 \text{ TeV})$  and with suppressed mixing with the ordinary  $Z$ -boson. Thus it decouples safely from electroweak physics. Another interesting aspect of supersymmetric  $U(1)'$  models that repeats here is that the tree-level light Higgs  $h_1$  mass exceeds its usual upper bound of  $m_Z$ . This is due to contributions from the  $U(1)'$   $D$ -terms to the quartic potential, which lift its otherwise flat direction. We find for its mass  $m_{h_1} \simeq 120 - 150 \text{ GeV}$  at tree level and  $m_{h_1} \simeq 150 - 180 \text{ GeV}$  at one loop.

A most interesting aspect of the  $U(1)'$  scenario is that the gauge-mediation scale is still the only fundamental scale, and the  $U(1)'$  scale is determined from it. It has been proposed recently<sup>44</sup> that perhaps the same  $U(1)'$  is also responsible for the actual mediation of supersymmetry breaking from the “hidden” sector to the messenger fields (*i.e.*, an “active”  $U(1)$  whose primary role is to mediate supersymmetry breaking). This is an ambitious yet interesting proposal that significantly differs from our bottom-up approach, which, in principle, is independent of the details of supersymmetry breaking and its initial mediation to the messenger fields. The  $U(1)$  discussed here is a “spectator” (rather than “active”)  $U(1)$  which does not participate in the supersymmetry breaking or mediation mechanisms. It witnesses supersymmetry breaking to the extent that the SM does (or even less so if the messenger fields are invariant under it). By distinguishing the two extended interactions

we avoid the need, e.g., to fine tune Yukawa couplings, which is the situation in Ref. 44 due to the multitude of tasks imposed there on a single  $U(1)$ . The only (moderate) hierarchy in Yukawa couplings that is assumed is between those that involve (exotic) quarks, which are taken to saturate or be near their infra-red quasi-fixed points and be  $\mathcal{O}(1)$ , and those which involve only the Higgs doublets and the singlet(s), which do not reach any (quasi-)fixed points and hence are taken to be smaller. Such differences naturally stem from QCD renormalization, which enables the existence of quasi-fixed points for the (exotic) quark couplings.

We now turn to describe a specific model in detail. The superpotential reads

$$W = -h_U H_U Q U + h_D H_D Q D + h_E H_D L E - \lambda_S S H_1 H_2 + \lambda_D S D_i D_i^c, \quad (18)$$

where the MSSM has been extended to include the singlet  $S$  and exotic quark vector-like pairs  $D$  and  $D^c$  which are singlets under  $SU(2)_L$  and carry hypercharge  $\pm 1/3$ . In Eq. (18) we include the usual Yukawa terms involving the third generation fields, an effective  $\mu$ -term  $\lambda_S S H_1 H_2$ , and a Yukawa coupling between the singlet and the exotic quark superfields. Given our assumptions, the free parameters in the analysis are  $\Lambda_M$ , the number  $n_D$  of  $D$ ,  $D^c$  pairs that couple to  $S$ , and the corresponding Yukawa couplings  $\lambda_D$ . ( $\lambda_S$  is fixed by the minimization condition (4).) The product of  $n_D$  and  $\lambda_D$  is constrained by electroweak breaking and also by requiring a sufficiently heavy  $Z'$ . Below we fix, as an example,  $n_D = 3$  and  $\lambda_D = 0.7$ . Following Eq. (2), the most general renormalizable potential involving the Higgs singlet field  $S$  is

$$V = |\lambda_S H_1 H_2|^2 + |\lambda_S S|^2 (|H_1|^2 + |H_2|^2) + \frac{G^2}{8} (|H_1|^2 - |H_2|^2)^2 + \frac{g^2}{2} |H_1^\dagger H_2|^2 + \frac{g_{1'}^2}{2} (Q_1 |H_1|^2 + Q_2 |H_2|^2 + Q_S |S|^2)^2 + m_{H_D}^2 |H_D|^2 + m_{H_U}^2 |H_U|^2 + m_S^2 |S|^2 + (A_s \lambda_S S H_1 H_2 + \text{H.c.}), \quad (19)$$

where  $G^2 = g_Y^2 + g^2$  and the the SSB terms are listed in last line.

The experimental constraint on the mass of the  $Z'$  can be satisfied if the  $U(1)'$  is broken at the TeV scale, which requires  $\langle H_D \rangle, \langle H_U \rangle \ll \langle S \rangle, \langle S' \rangle$ . This separation is indeed realized in our example and the determination of the  $S$  VEV can therefore be separated to a very good approximation from that of the Higgs VEV's. The scalar potential for  $S$  then reads

$$V = m_S^2 |S|^2 + \frac{g_{1'}^2}{2} (Q_S |S|^2)^2. \quad (20)$$

It acquires a VEV  $\langle S \rangle = s/\sqrt{2}$  where

$$s^2 = -\frac{2m_S^2}{g_1^2 Q_S^2}, \quad (21)$$

if the evolution of  $m_S^2$  can be neglected near the minimum. Hence, a large value for  $s$  occurs for  $m_S^2$  large and negative. This is achieved by the order unity Yukawa couplings between  $S$  and exotic quark pairs  $D$  and  $D^c$  (with scalar mass-squares  $m_{D, D^c}^2(\Lambda_M) \gg m_S^2(\Lambda_M) \simeq 0$ ), which rapidly diminish  $m_S^2(Q < \Lambda_M)$  via the usual renormalization group evolution. The mass of the  $Z'$  boson, which is independent of  $g_1$ , is

$$m_{Z'} \sim g_1 Q_S s \sim \sqrt{2|m_S^2|}, \quad (22)$$

with the  $Z - Z'$  mixing angle  $\alpha_{Z-Z'} = \mathcal{O}(m_Z^2/m_{Z'}^2)$ . The  $Z'$  mass and the  $U(1)'$  scale are determined by the only scale in the problem,  $\Lambda_M$  (which is encoded in  $m_S^2$ ). The VEV of  $S$  generates an effective  $\mu$ -parameter  $\mu = \lambda_S s/\sqrt{2}$ . The  $A$ -term associated with  $SH_D H_U$ , which is non-zero at the electroweak scale due to gluino loop corrections, generates an effective  $m_{H_{UD}}^2$  for the two Higgs doublets  $m_{H_{UD}}^2 = A_s \mu$ . (In addition, the  $U(1)'$   $D$ -term generates corrections to the Higgs scalar masses  $\delta m_{H_{D,U}}^2 = (g_1^2/2)Q_{1,2}Q_S s^2$ .)

As an example of actual derivation of RSB and the spectrum, let us list the model spectrum at the infra-red  $\sim \Lambda_W$  in the case of the  $E_6 U(1)_\eta$  assignments  $Q_1 = 1$ ,  $Q_2 = 4$  and  $Q_S = -Q_1 - Q_2$ ,  $\Lambda_M = 10^5$  GeV, three pairs of exotic quark singlets, and  $\lambda_D(\Lambda_M) = 0.7$ . (For full listing and other examples, see Ref. 41.) The VEV of the singlet is  $s = 3720$  GeV, the VEV's of the Higgs doublets are  $\langle H_D^0 \rangle = 14$  GeV and  $\langle H_U^0 \rangle = 245$  GeV, resulting in a solution with  $\mu = 1050$  GeV (or equivalently,  $\lambda_S(\Lambda_M) = 0.47$ ) and  $\tan \beta = 18$ . The effective  $m_{H_{UD}}^2$  is  $\sim (235 \text{ GeV})^2$ . The  $Z'$  mass is  $m_{Z'} = 1110$  GeV and the  $Z - Z'$  mixing angle is  $\alpha_{Z-Z'} = 0.004$ . The (tree-level) spectrum of the CP even physical Higgs is  $m_{h_1} = 124$  GeV,  $m_{h_2} = 995$  GeV,  $m_{h_3} = 1090$  GeV, while  $m_{h_4} = 154$  GeV at one loop (with negligible corrections to  $m_{h_{2,3}}$ ). The CP odd Higgs scalar and the charged Higgs masses are  $m_A \simeq m_{H^\pm} = 993$  GeV. The heaviest CP even Higgs scalar  $h_3$  is mainly composed of the singlet  $S$ , associated with the breaking of the  $U(1)'$ . The second heaviest CP even Higgs, the CP odd Higgs and the charged Higgs fields form the  $SU(2)$  doublet that is not associated with the  $SU(2) \times U(1)_Y$  breaking. The masses of the two charginos are  $m_{\tilde{\chi}_1^\pm} = 266$  GeV and  $m_{\tilde{\chi}_2^\pm} = 1060$  GeV. The lightest (heaviest) chargino is predominantly a gaugino (Higgsino). The spectrum of the neutralinos is  $m_{\tilde{\chi}_1^0} = 142$  GeV,  $m_{\tilde{\chi}_2^0} = 266$  GeV,  $m_{\tilde{\chi}_3^0} = 1060$  GeV,  $m_{\tilde{\chi}_4^0} = 1060$  GeV,  $m_{\tilde{\chi}_5^0} = 1120$  GeV,  $m_{\tilde{\chi}_6^0} = 1120$  GeV. In the limit of



neglecting the Higgs VEVs, the two lightest neutralinos are just  $\tilde{B}$  and  $\tilde{W}_3$ , *i.e.*, the Bino and the Wino.  $\tilde{\chi}_{3,4}$  are linear combinations of Higgsinos with nearly degenerate masses  $\sim \mu = h_s s / \sqrt{2}$ ; and  $\tilde{\chi}_{5,6}$  are linear combinations of the other gaugino  $\tilde{B}'$  and the singletino  $\tilde{S}$  with degenerate masses  $\sim m_{Z'}$ . Squark and gluino masses are in the 1200 – 1400 GeV range. The next to lightest sparticle (NLSP) is the lightest neutralino, which is predominantly the bino, *i.e.*, the gaugino of the  $U(1)_Y$ .

We note in passing the usual near equality between  $|\mu|$  and the gluino mass that often appears in various variants of the MSSM. (See Sec. 2 for a discussion.) The gluino is heavy, which is a generic prediction of the gauge-mediation framework, and hence it naturally leads to a relatively large value of  $\mu$ . In particular, there is no new tuning due to the  $U(1)'$  dynamics. The fine-tuning question can be phrased in this case in terms of the particular value of  $\lambda_S$ . It is worth stressing that the Higgs mixing parameter in the scalar potential  $m_{H_{UD}}^2 = \mu A_s$  is a geometrical mean of the superpotential Higgs mixing parameter  $\mu$  and a radiatively generated (small) trilinear coupling  $A_s$ . Since  $\mu$  is proportional to a large VEV (and the heavy gluino implies further that  $|\mu|$  is not suppressed by a small coupling) the geometrical mean  $m_{H_{UD}} \sim \text{a few} \times 100$  GeV is sufficiently large.

In conclusion, the Higgs mass parameters in the gauge-mediation framework are best understood as dynamical degrees of freedom corresponding to a (SM) singlet. Here, it was suggested that such a singlet is not a gauge singlet but transforms under a  $U(1)'$ . The  $U(1)'$  scale may be generated naturally and radiatively one or two orders of magnitude below the messenger scale. Upon integrating out the  $U(1)'$  sector, the supersymmetry conserving ( $\mu$ ) and breaking ( $m_{H_{UD}}^2$ ) dimensionful Higgs mixing parameters are generated, resolving the  $\mu$  problem in the otherwise attractive class of gauge-mediation models. The  $U(1)'$  dynamics also adds to the already strong predictive power of the gauge-mediation framework, as the scalar, fermion and vector electroweak sectors are extended and new exotic matter is predicted at a few TeV scale. Further discussion and examples can be found in Ref. 41.

## 5 Generalization to Lepton Number Violation

Before concluding, let us note that the MSSM contains three more gauge-invariant holomorphic bilinear terms,  $L_i H_U$   $i = 1, 2, 3$ , which could appear in the superpotential or the scalar potential, corresponding to lepton number violating (LNV)  $\mu$  and  $m_{H_{UD}}^2$  (or  $B$ ) terms. In the MSSM one imposes the conservation of  $R$ -parity,  $R_P = (-)^{3B+L+2S}$ , which encompasses both lepton  $L$  and baryon  $B$  number conservation (and  $S = 0, 1/2, 1$  is the particle spin).

This is sufficient but not necessary to ensure proton stability. LNV terms could be admitted as long as baryon number is (sufficiently) conserved! It was proposed<sup>45</sup> that  $W_{\text{LNV}} \sim \mu_L L H_U$  could therefore be present and provide an electroweak scale source for neutrino masses and mixing.

This brings us back to the operators (9). By interplay of symmetries (which distinguish Higgs and lepton fields in the ultra-violet theory) and  $F$ - and  $D$ -type operators in Eq. (9), one can realize simultaneously both the usual and LNV  $\mu$ -terms once integrating out the background fields  $X$ .<sup>13</sup> They could be comparable in size or maintain a certain hierarchy. This offers, on the one hand, a minimal LNV extension of the MSSM, while on the other hand, already contains an explanation to its minimality: If LNV arises from non-renormalizable operators suppressed by some scale  $M$ , then on dimensional grounds LNV Yukawa couplings  $h_{\text{LNV}} \sim \mu_{\text{LNV}}/M \rightarrow 0$ , leaving the bilinear term as the primary source of LNV. (Another possible explanation will be offered below.)

In theories in which the SSB parameters do not distinguish  $H_D$  from the (s)lepton fields (Higgs-lepton universality), it is straightforward to show that in the infra-red, after minimizing the potential and redefining the (MSSM) Higgs  $H_D$  along the VEV (in a four dimensional field space spanned by  $H_D$  and the three lepton doublets),  $|\mu|$  is again concentrated in the usual Higgs mixing term and there is no LNV in the effective tree-level theory. However, quantum corrections proportional to the usual Yukawa couplings spoil the Higgs-lepton universality, and therefore a small LNV remains in the renormalized theory and it controls the size of the effective LNV Yukawa couplings (which are given by the usual Yukawa couplings times a rotation angle) as well as the Higgsino-neutrino mixing, and hence the neutrino spectrum. This is the dynamical alignment mechanism<sup>13,46</sup> which asserts that given Higgs-lepton universality the size of the neutrino mass arising at tree-level from Higgsino-neutrino mixing is suppressed by the dynamical alignment between the  $\mu$  and VEV vectors in the relevant four-dimensional field space. The alignment itself arises trivially upon minimization as the  $\mu$  vector defines the only direction in field space; hence, dynamical alignment. The extent of the alignment is determined by the relative size of the radiative corrections to the relevant SSB parameters, which spoil the alignment. These corrections can be calculated in a given model and typically one finds neutrino masses of the order of MeV or smaller. In fact, one can formulate these arguments in terms of a chiral  $SU(4)$  symmetry controlling the Higgsino-neutrino mixing which is broken at tree level by the fundamental  $\mu$  and VEV four-vectors to either  $SU(3)$ , if the alignment holds (massive Higgsino but massless neutrino), or  $SU(2)$  otherwise (massive Higgsino and one massive neutrino).

Having discussed a gauged NMSSM model in Sec. 4.2, we note that it is straightforward to extend the discussion there to include bilinear LNV through couplings  $h_{\psi}SLH_U \rightarrow \mu_{LNV}LH_U$  if  $L$  and  $H_D$  carry the same  $U(1)'$  charge (this is the case for a  $U(1)_\eta$  of  $E_6$  embedding), or more generally in a multi-singlet model. The case  $Q_{H_D} \neq Q_L$  is in fact more attractive since it would forbid lepton number violating Yukawa operators in the high-energy theory. Since gauge mediation guarantees Higgs-slepton mass universality, and Higgs-slepton bilinear mixing in the scalar potential arises only from radiative  $A$ -parameters, then all conditions for the dynamical alignment suppression of neutrino masses are automatically and naturally satisfied. Hence, such an extension provides another realization of the bilinear LNV framework.

LNV Yukawa couplings appear in the infra-red as a result of the redefinition of the Higgs field along the VEV. As mentioned above, they are proportional to the usual Yukawa matrices, which leads to a clear prediction that the LNV Yukawa couplings in  $W \sim \lambda_{i33}L_iQ_3D_3$  are proportional to  $b$ -quark mass, and hence are the dominant LNV Yukawa couplings. (This can have a significant effect on the corresponding collider phenomenology,<sup>47</sup> for example.) Since the usual Yukawa couplings do not respect the  $SU(4)$  symmetry (and hence lead to the radiative corrections to the alignment mentioned below), then the LNV Yukawa couplings, which are proportional to the usual ones, also break (maximally) the  $SU(4)$  symmetry and lead to finite quantum corrections to the neutrino masses so that all neutrinos are massive at the quantum level. This results in a variety of models and neutrino mass patterns. It is a miraculous complementarity between the SM where the neutrino must remain massless and the MSSM where only the neutrinos can have an explicit mass term. For that matter, Higgs and lepton fields are not distinguished by the symmetries of the model, unless one imposes such a distinction. It is therefore also crucial that one can naturally generate a large hierarchy between the Higgsino and neutrino masses. Phenomenology and viability of these models was studied extensively<sup>48</sup> over the last few years.

## 6 Summary

In summary, we have demonstrated the phenomenological correlation between the size of the supersymmetry conserving  $\mu$  parameter and the SSB parameters and established rough lower and upper bounds for  $|\mu|$ . The  $\mu$  puzzle was then defined as the question of the origin of this correlation, which also suggests that the Higgs fields may be distinguished from all other matter in the way that they communicate with heavy and supersymmetry breaking fields (labeled as the background fields  $X$  in this lecture). In order to address this

puzzle in some generality we pursued an operator analysis which was then applied to different frameworks for the SSB parameters, comparing all possibilities on equal footing. While in models with  $m_{3/2} \sim \Lambda_W$  the correlation could naturally arise from various sources, in models where the SSB parameters and the weak scale are determined by (gauge or gravity) quantum corrections the puzzle transformed into a new hierarchy problem between the Higgs mixing in the superpotential and the scalar potential. Two specific frameworks for the solution, both based on unconventional variants of the NMSSM, were then presented in some detail, the first assuming gravity mediation and the second assuming gauge mediation of the SSB parameters. Lastly, we also noted that (i)  $\mu$  may be a SSB parameter in certain situations and (ii) that it is straightforward to admit Higgs-lepton mixing which generalizes the usual  $\mu$ -term and leads to neutrino masses and mixing. The  $\mu$ -parameter and its mysterious origin have fueled many works in recent years. Here we attempted to catalog some of those ideas and proposals, and elaborated on two more recent ones. To conclude, the discovery of supersymmetry would lead to the measurement of  $\mu$ , and the information it encodes could be unfolded. The measurement and its interpretation will then open a new window to the ultra-violet.

I would like to thank Professor J. E. Kim for his kind invitation and the Korea Institute for Advanced Study and the Center for Theoretical Physics at Seoul National University for their hospitality. Work supported by the US Department of Energy under cooperative research agreement No. DE-FC02-94ER40818. The support of Rutgers University during earlier parts of this research is acknowledged.

## References

1. Report of SUGRA Working Group for RUN II of the Tevatron, S. Abel *et al.*, URL <http://fnth37.fnal.gov/sugra.html>.
2. F. Borzumati, G.R. Farrar, N. Polonsky and S. Thomas, *Nucl. Phys. B* **555**, 53 (1999).
3. J.E. Kim and H.P. Nilles, *Phys. Lett.* **138B**, 150 (1984).
4. For a discussion in this spirit, see, for example, R. Barbieri, M. Frigeni and F. Caravaglios, *Phys. Lett. B* **258**, 167 (1991); H.E. Haber and R. Hempfling, *Phys. Rev. D* **48**, 4280 (1993).
5. M. Drees, M. Gluck and K. Grassie, *Phys. Lett. B* **164**, 157 (1985); P. Langacker and N. Polonsky, *Phys. Rev. D* **50**, 2199 (1994).
6. P. Langacker and N. Polonsky, *Phys. Rev. D* **47**, 4028 (1993); *Phys. Rev. D* **52**, 3081 (1995); M. Carena, S. Pokorski and C.E. Wagner, *Nucl. Phys. B* **406**, 59 (1993).

7. L.J. Hall and L. Randall, *Nucl. Phys. B* **352**, 289 (1991). L. Randall and N. Rius, *Phys. Lett. B* **286**, 299 (1992).
8. J.L. Feng, N. Polonsky and S. Thomas, *Phys. Lett. B* **370**, 95 (1996).
9. For a model-dependent analysis in this spirit, see J. Ellis, T. Falk, K.A. Olive and M. Schmitt, *Phys. Lett. B* **388**, 97 (1996). For a recent related discussion, see T. Falk, *Phys. Lett. B* **456**, 171 (1999).
10. R. Arnowitt, in this volume, and references therein.
11. W. Buchmuller and D. Wyler, *Phys. Lett.* **121B**, 321 (1983).
12. J.E. Kim and H.P. Nilles, *Mod. Phys. Lett. A* **9**, 3575 (1994); K. Choi, E.J. Chun and H. Kim, *Phys. Rev. D* **55**, 7010 (1997); J.E. Kim, *Phys. Lett. B* **452**, 255 (1999).
13. H. Nilles and N. Polonsky, *Nucl. Phys. B* **484**, 33 (1997).
14. E.J. Chun, J.E. Kim and H.P. Nilles, *Nucl. Phys. B* **370**, 105 (1992); E.J. Chun, *Phys. Lett. B* **348**, 111 (1995). For a discussion in the context of gauge mediation, see E.J. Chun, *Phys. Rev. D* **59**, 015011 (1999).
15. See, for example, S. Abel, S. Sarkar and P. White, *Nucl. Phys. B* **454**, 663 (1995); U. Ellwanger, M. Rausch de Traubenberg and C. Savoy, *Nucl. Phys. B* **492**, 21 (1997); and references therein.
16. Y. Nir, *Phys. Lett. B* **354**, 107 (1995); V. Jain and R. Shrock, hep-ph/9507238.
17. D. Suematsu and Y. Yamagishi, *Int. J. of Mod. Phys. A* **10**, 4521 (1995); M. Cvetič and P. Langacker, *Phys. Rev. D* **54**, 3570 (1996); *Mod. Phys. Lett. A* **11**, 1247 (1996).
18. R.N. Mohapatra, "Supersymmetric Grand Unification," hep-ph/9801235.
19. L. Hall, J. Lykken and S. Weinberg, *Phys. Rev. D* **27**, 2359 (1983).
20. I. Antoniadis, E. Gava, K.S. Narain and T.R. Taylor, *Nucl. Phys. B* **432**, 187 (1994).
21. C. Kolda, S. Pokorski and N. Polonsky, *Phys. Rev. Lett.* **80**, 5263 (1998).
22. J.A. Casas and C. Munoz, *Phys. Lett. B* **306**, 288 (1993).
23. K. Choi, J.S. Lee and C. Munoz, *Phys. Rev. Lett.* **80**, 3686 (1998).
24. G. Giudice and A. Masiero, *Phys. Lett. B* **206**, 480 (1988).
25. M. Drees, *Phys. Rev. D* **33**, 1468 (1986); S. Dimopoulos and G. F. Giudice, *Phys. Lett. B* **357**, 573 (1995); A. Pomarol and D. Tommasini, *Nucl. Phys. B* **466**, 3 (1996); A. G. Cohen, D. B. Kaplan and A. E. Nelson, *Phys. Lett. B* **388**, 588 (1996).
26. For an example of a model, see G. Dvali and A. Pomarol, *Phys. Rev. Lett.* **77**, 3728 (1996); *Nucl. Phys. B* **522**, 3 (1998). (The  $\mu$ -term is discussed explicitly in the second reference.)
27. J.L. Feng, C. Kolda and N. Polonsky, *Nucl. Phys. B* **546**, 3 (1999); J.A. Bagger, J.L. Feng and N. Polonsky, hep-ph/9905292, *Nucl. Phys.*

- B, in press, J.A. Bagger, J.L. Feng, N. Polonsky and R. Zhang, hep-ph/9911255.
28. H.-P. Nilles and N. Polonsky, *Phys. Lett. B* **412**, 69 (1997).
  29. K.S. Babu, C. Kolda and F. Wilczek, *Phys. Rev. Lett.* **77**, 3070 (1996); F.M. Borzumati, hep-ph/9702307; R. Rattazzi and U. Sarid, *Nucl. Phys. B* **501**, 297 (1997).
  30. G. Dvali, G.F. Giudice and A. Pomarol, *Nucl. Phys. B* **478**, 31 (1996).
  31. L. Randall and R. Sundrum, hep-th/9810155; J.A. Bagger, T. Moroi and E. Poppitz, hep-th/9911029.
  32. A. Pomarol and R. Rattazzi, *JHEP* **05**, 013 (1999); E. Katz, Y. Shadmi and Y. Shirman, *JHEP* **08**, 015 (1999); Z. Chacko, M.A. Luty, I. Maksymyk and E. Ponton, hep-ph/9905390; K. Choi, N. Polonsky and Y. Shirman, work in progress.
  33. J. Bagger and E. Poppitz, *Phys. Rev. Lett.* **71**, 2380 (1993).
  34. Borzumati *et al.*, Ref. 2; I. Jack and D.R. Jones, *Phys. Lett. B* **457**, 101 (1999); hep-ph/9909570; S.P. Martin, hep-ph/9907550; N. Polonsky, “Non-standard supersymmetry breaking terms: Implications and applications” Talk presented at SUSY ‘99, Fermilab, Batavia, IL, June 14 – 19, 1999 (unpublished).
  35. K. Choi and H.D. Kim, hep-ph/9906363.
  36. H.-P. Nilles, M. Srednicki and D. Wyler, *Phys. Lett.* **124B**, 337 (1983); A. Lahanas, *Phys. Lett.* **124B**, 341 (1983); U. Ellwanger, *Phys. Lett.* **133B**, 187 (1983).
  37. V. Jain, *Phys. Lett. B* **351**, 481 (1995).
  38. J. Bagger, E. Poppitz and L. Randall, *Nucl. Phys. B* **455**, 59 (1995).
  39. Z. Chacko and R.N. Mohapatra, *Phys. Rev. D* **58**, 015001 (1998); R.N. Mohapatra, private communications.
  40. T. Yanagida, *Phys. Lett. B* **400**, 109 (1997).
  41. P. Langacker, N. Polonsky and J. Wang, *Phys. Rev. D* **60**, 115005 (1999).
  42. M. Dine and A.E. Nelson, *Phys. Rev. D* **48**, 1277 (1993); K. Agashe and M. Graesser, *Nucl. Phys. B* **507**, 3 (1997); A. de Gouvea, A. Friedland and H. Murayama, *Phys. Rev. D* **57**, 5676 (1998). For a more recent proposal addressing the domain wall problem, see T. Han, D. Marfatia and R.-J. Zhang, hep-ph/9906508.
  43. T. Moroi, *Phys. Lett. B* **447**, 75 (1999).
  44. H.-C. Cheng, B.A. Dobrescu and K.T. Matchev, *Phys. Lett. B* **439**, 301 (1998); *Nucl. Phys. B* **543**, 47 (1999).
  45. L. J. Hall and M. Suzuki, *Nucl. Phys. B* **231**, 419 (1984).
  46. R. Hempfling, *Nucl. Phys. B* **478**, 3 (1996).
  47. See, for example, J. Erler, J.L. Feng and N. Polonsky, *Phys. Rev.*

- Lett.* **78**, 3062 (1997); F. Borzumati, J.-L. Kneur and N. Polonsky, hep-ph/9905443, *Phys. Rev. D*, in press, and references therein.
48. See, for example, A.S. Joshipura and M. Nowakowski, *Phys. Rev. D* **51**, 2421 (1995); B. Mukhopadhyaya, S. Roy and F. Vissani, *Phys. Lett. B* **443**, 191 (1998); M. Bisset, O.C. Kong, C. Macesanu and L.H. Orr, hep-ph/9811498; J.C. Romao, M.A. Diaz, M. Hirsch, W. Porod and J.W. Valle, hep-ph/9907499; E.J. Chun and S.K. Kang, hep-ph/9909429; F. Takayama and M. Yamaguchi, hep-ph/9910320; and references therein.